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The non-abelian tensor multiplet in loop space

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Abstract: We introduce a non-abelian tensor multiplet directly in the loop space associated with flat six-dimensional Miskowski space-time, and derive the supersymmetry variations for on-shell $\mathcal{N} = (2, 0)$ supersymmetry.

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Contents

1. Introduction

The highest dimension in which one can have a superconformally invariant theory is $d = 6$ [2] and the maximally supersymmetric theory in $d = 6$ has $\mathcal{N} = (2,0)$ chiral supersymmetry. The more symmetries we require on our theory, the better its quantum behaviour. One might hope that these maximally supersymmetic theories in six dimensions will enjoy the same finiteness property as their close relatives in four dimensions, $\mathcal{N} = 4$ super Yang-Mills. Due to the difficulties with quantizing gravity, it has even been suggested that the $(2,0)$ theory might be 'the theory of everything' [7]. According to that picture our universe would be a curved[1] three brane embedded in flat six dimensions. Indeed the $(2,0)$ supersymmetry algebra allows for a central extension that involves a three brane (as well as a selfdual string) [8]. Although this is just a speculation, it calls for a further investigation of the $(2,0)$ theories.

But it is problematic to quantize $(2,0)$ theory. The coupling constant is a fixed number ~ 1 due to self-duality and the dyonic charge quantization condition for strings in six dimensions. It may therefore not be possible to go from a classical theory to a quantum perturbation theory. It is possibly that $(2,0)$ theory only exists as a quantum theory, with no classical limit. But one way to obtain a related quantum theory would be if one could find solitonic solutions to some classical equations of motion. One should then be able to find a quantum theory by expanding quantum fields about this classical solution in a parameter which is related to the inverse tension of the extended object.

In this Letter we will indeed derive the classical equations of motion, though in loop space. We will introduce a non-abelian tensor multiplet in loop space, and show that it closes the supersymmetry algebra on-shell and as a by product get the non-abelian equations of motion of the loop fields in the tensor multiplet.

[1] In that way we get an induced gravity.

JHEP01(2006)165

It thus appears to be the unique way in which to generalize the abelian tensor multiplet. But whether our results will find any practical use is unclear. We do not know how to handle equations in loop space, and we do not think that it is obvious how to descend from loop fields to local fields. We will mention a few difficulties that we encountered when we tried this, in the last section.

2. The tensor multiplet and its constraints in loop space

We will assume flat $d = 1 + 5$ dimensional Minkowski space-time M with metric tensor $\eta_{\mu\nu}$ =diag$(-1, 1, 1, 1, 1, 1)$ and Lorentz symmetry group SO(1, 5). The (2, 0)-supersymmetry is generated by 16 real supercharges transforming in the chiral representation $(4, 4)$ of $SO(1, 5) \times SO(5)$, where SO(5) is an internal R-symmetry group. Our spinor conventions are the same as in [3], and these are collected in appendix. Requiring all this supersymmetry and no dynamical gravity, there is just one abelian multiplet, namely the tensor multiplet. It consists of a two-form gauge potential $B_{\mu\nu}(x)$ with anti self-dual field strength $H_{\mu\nu\rho}(x) = -\frac{1}{6}\epsilon_{\mu\nu\rho\kappa\tau\sigma}H^{\kappa\tau\sigma}(x)$, five Lorentz scalars $\phi^A(x)$ (where A is a vector index of SO(5)), and four real chiral (i.e. symplectic Majorana-Weyl) spinors $\psi(x)$ which transform in the same $(4, 4)$-representation as the supercharges.

An abelian two-form gauge potential $B_{\mu\nu}(x)$ in M can alternatively be viewed in a parametrized loop space as a one-form,

$$A_\mu(C) := \int ds \dot{C}^\nu(s) B_{\nu\mu}(C(s)). \tag{2.1}$$

Here C denotes a parametrized loop $s \mapsto C^\mu(s)$ in M and s will always run over some fixed interval, say $s \in [0, 2\pi]$. In [6] we also introduced abelian loop fields corresponding to the other fields in the abelian tensor multiplet,

$$\begin{aligned}\phi^A_\mu(C) &:= \int ds \dot{C}_\mu(s)\phi^A(C(s)) \\ \psi_\mu(C) &:= \int ds \dot{C}_\mu(s)\psi(C(s)).\end{aligned} \tag{2.2}$$

In the non-abelian case we suggested in [6] the following representation for the loop fields

$$A^A_\mu(C) = \int ds A^a_\mu(s, C)\lambda_a(s) \tag{2.3}$$

and similarly for the other fields in the tensor multiplet, where $\lambda_a(s)$ denote generators of the loop algebra associated with the gauge group with structure constants $C_{ab}{}^c$, that is,

$$[\lambda_a(s), \lambda_b(t)] = C_{ab}{}^c\delta(s - t)\lambda_c(s). \tag{2.4}$$

It should be noticed that we are not giving a very concrete representation here of the loop fields. Apriori $A_\mu(s, C)$ may depend in any non-local way on the loop C. There also exists a more concrete way to represent loop fields in terms of a local connection one-form and two-form (see for instance [4]). We have therefore aimed to keep our discussion completely general in this and the next sections by not specifying a representatation of the loop fields.

JHEP01(2006)165

We begin with the abelian case, and then look for a natural non-abelian generalization. We introduce a derivative in loop space,

$$\partial_\mu(C) := \int ds \frac{\delta}{\delta C^\mu(s)} \tag{2.5}$$

and a gauge covariant derivative

$$D_\mu(C) := \partial_\mu(C) + A_\mu(C) \tag{2.6}$$

though in the sequel we will drop the arguments C where it should always be obvious from the context whether ∂_μ denotes a derivative in loop space or in space-time. The gauge covariant field strength is $F_{\mu\nu} = [D_\mu, D_\nu]$. The gauge transformations act as (considering infinitesimal transformations generated by the loop field $\Lambda(C)$),

$$\begin{aligned} \delta A_\mu &= D_\mu \Lambda \\ \delta F_{\mu\nu} &= [F_{\mu\nu}, \Lambda] \\ \delta \phi_\mu^A &= [\phi_\mu^A, \Lambda] \\ \delta \psi_\mu &= [\psi_\mu, \Lambda] \end{aligned} \tag{2.7}$$

We are now ready to write down the constraints on the abelian loop fields. They are

$$\begin{aligned} \partial^\mu \phi_\mu^A &= 0 \\ \partial^\mu \psi_\mu &= 0 \end{aligned} \tag{2.8}$$

which is easily seen by computing

$$\partial_\nu(C)\phi_\mu^A(C) = \int ds \dot{C}_\mu(s)\partial_\nu \phi^A(C(s)) - \eta_{\mu\nu} \int ds \dot{C}^\kappa(s)\partial_\kappa \phi^A(C(s)). \tag{2.9}$$

We then see that $\partial^\mu \phi_\mu^A$ corresponds to a total derivative which vanishes when integrated over the loop.

How should these constraints be generalized to the non-abelian case?[2] The natural generalization is of course to take the following gauge covariant non-abelian constraints,

$$\begin{aligned} D^\mu \phi_\mu^A &= 0 \\ D^\mu \psi_\mu &= 0 \end{aligned} \tag{2.10}$$

But now it is not consistent with supersymmetry to impose these constraints alone, without also imposing the constraint

$$[\phi_\mu^A, \psi_\nu] = [\phi_\nu^A, \psi_\mu] \tag{2.11}$$

To see this, we impose the following supersymmetry variations of the Bose loop fields,

$$\delta \phi_\mu^A = -i\bar{\epsilon}\Gamma^A \psi_\mu$$

[2]Using the representation (2.1) it is easy to see that the above constraints can not persist to the non-abelian case. The loop derivative will hit (in a subtle way) on the generators $\lambda^a(s)$ as well.

$$\delta A_\mu = -i\bar{\epsilon}\Gamma_{\kappa\mu}\psi^\kappa \tag{2.12}$$

and find that the supersymmetry variation of constraint becomes

$$\Gamma^A D^\mu \psi_\mu + \Gamma^{\nu\mu}[\psi_\nu, \phi^A_\mu] = 0. \tag{2.13}$$

Hence we see that supersymmetry implies that we must also impose the constraint (2.11).[3] We should also impose the constraint

$$[\phi^A_{[\mu}, \phi^B_{\nu]}] = 0. \tag{2.15}$$

Since the Fermi field ψ_μ thus is constrained, we introduce the somewhat simpler field

$$\psi := \Gamma^\mu \psi_\mu \tag{2.16}$$

with no vector index, for which we find the relations

$$[\psi, \phi^A_\nu] = \Gamma^\mu[\psi_\nu, \phi^A_\mu] \tag{2.17}$$

and

$$\Gamma^\nu[\psi, \phi^A_\nu] = [\psi^\nu, \phi^A_\nu]. \tag{2.18}$$

3. $\mathcal{N} = (2, 0)$ supersymmetry

We are now ready to construct the full supersymmetry transformations. We have already specified the variations of the bosonic fields. We will also need the variation of the field strength,

$$\delta F_{\mu\nu} = 2i\bar{\epsilon}\Gamma_{\kappa[\mu}D_{\nu]}\psi^\kappa \tag{3.1}$$

We now make the most general ansatz compatible with Poincare invariance and dimensional analysis for this fermi field, which is such that it reduces to the known Abelain transformation if we take the gauge group to be abelian,

$$\delta_\epsilon\psi = \Big(\frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu} + D_\mu\phi^A_\nu\left(\Gamma^{\nu\mu} + a\eta^{\mu\nu}\right)\Gamma_A + \frac{1}{2}\left[\phi^A_\mu, \phi^B_\nu\right]\left(c\Gamma^{\mu\nu}\delta_{AB} + d\eta^{\mu\nu}\Gamma_{AB}\right)\Big)\epsilon \tag{3.2}$$

but, noting the constraints, we directly see that we can reduce this ansatz to just

$$\delta_\epsilon\psi = \left(\frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu} + D_\mu\phi^A_\nu\Gamma^{\nu\mu}\Gamma_A + \frac{d}{2}\left[\phi^A_\mu, \phi^B_\nu\right]\eta^{\mu\nu}\Gamma_{AB}\right)\epsilon \tag{3.3}$$

[3] It is also easy to see this constraint directly. We compute

$$\begin{aligned}[]
[\phi^A_\mu, \psi_\nu] &= \int ds \int dt \dot{C}_\mu(s)\dot{C}_\nu(t)[\phi^A(C(s)), \psi(C(t))] \\
&= C_{ab}{}^c \int ds \dot{C}_\mu(s)\dot{C}_\nu(s)\phi^{A,a}(C(s))\psi^b(C(s))\lambda_c(s)
\end{aligned} \tag{2.14}$$

and see that this is manifestly symmetric in $(\mu\nu)$.

JHEP01(2006)165

We begin with computing the commutor of two supersymmetry variations when acting on the Fermi loop field ψ, saving the bosonic fields for later;

$$
\begin{aligned}
[\delta_\eta, \delta_\epsilon]\,\psi = \; & i\Gamma^{\mu\nu}\left(\epsilon\bar{\eta} - \eta\bar{\epsilon}\right)\Gamma_\mu D_\nu \psi \\
& + i\Gamma^{\mu\nu}\left(\epsilon\bar{\eta} - \eta\bar{\epsilon}\right) D_\mu \psi_\nu \\
& + i\Gamma^{\mu\nu}\Gamma_A\left(\epsilon\bar{\eta} - \eta\bar{\epsilon}\right)\Gamma^A D_\mu \psi_\nu \\
& + i\Gamma^{\nu\mu}\Gamma_A\left(\epsilon\bar{\eta} - \eta\bar{\epsilon}\right)\Gamma_\mu\left[\psi, \phi^A_\nu\right] \\
& - id\Gamma_{AB}\left(\epsilon\bar{\eta} - \eta\bar{\epsilon}\right)\Gamma^A\left[\psi^\mu, \phi^B_\mu\right]
\end{aligned} \tag{3.4}
$$

Here we have made use of various constraints. Then using a Fierz rearrangement and various gamma matrix identities (which we have collected in the appendix), we get

$$
\begin{aligned}
[\delta_\eta, \delta_\epsilon]\,\psi = & -\frac{2i}{16}(\bar{\eta}\Gamma_\eta\epsilon)\Big\{16D^\eta\psi - 8\Gamma^\mu D_\mu\psi^\eta + \Gamma^\eta\Gamma^\nu D_\nu\psi - 4\Gamma^\eta D^\mu\psi_\mu \\
& \qquad - 8\Gamma_A[\psi, \phi^{A,\eta}] - (4d-3)\Gamma_A\Gamma^\eta\Gamma^\nu[\psi, \phi^A_\nu]\Big\} \\
& + \frac{2i}{16}(\bar{\eta}\Gamma_\eta\Gamma_C\epsilon)\Big\{\Gamma^C\left(8\Gamma^\mu D_\mu\psi^\eta + \Gamma^\eta\Gamma^\nu D_\nu\psi - 4\Gamma^\nu D^\mu\psi_\mu\right) \\
& \qquad + \Gamma^C\Gamma_A\left(8[\psi, \phi^{A,\eta}] + (3-2d)\Gamma^\eta[\psi^\mu, \phi^A_\mu]\right) - 16[\psi, \phi^{C,\eta}]\Big\} \\
& - \frac{i}{192}(\bar{\eta}\Gamma_{\eta\omega\tau}\Gamma_{CD}\epsilon)\Big\{\Gamma^{CD}\Gamma^{\eta\omega\tau}\left(-\Gamma^\nu D_\nu\psi + \Gamma^\nu\Gamma_A[\psi, \phi^A_\nu]\right) \\
& \qquad + 4(1-d)\delta^{[C}_A\Gamma^{D]}\Gamma^{\eta\omega\tau}\Gamma^\nu[\psi, \phi^A_\nu]\Big\}
\end{aligned} \tag{3.5}
$$

For this to become a representation of the $(2,0)$-supersymmetry algebra, $[\delta_\eta, \delta_\epsilon] = -2i\left(\bar{\eta}\Gamma^\nu\epsilon\right)\partial_\nu$ (modulo a gauge transformation), we must take $d = 1$ and the Fermi equation of motion to be

$$
\Gamma^\nu\left(D_\nu\psi + \Gamma_A[\phi^A_\nu, \psi]\right) = 0. \tag{3.6}
$$

To proceed with the Bose loop fields we need also the variation of ψ^μ. It is easy to see that the following variation

$$
\delta\psi^\mu = \left(\frac{1}{12}G^\mu{}_{\tau\rho\sigma}\Gamma^{\tau\rho\sigma} + D_\nu\phi^{A,\mu}\Gamma^\nu\Gamma_A + \frac{d}{2}\left[\phi^{\mu,A}, \phi^B_\nu\right]\Gamma^\nu\Gamma_{AB}\right) \tag{3.7}
$$

implies the above variation of ψ provided

$$
G^\kappa{}_{\kappa\rho\sigma} = F_{\rho\sigma} \tag{3.8}
$$

$$
\frac{1}{6}\epsilon_{\mu\sigma\kappa}{}^{\tau\rho\eta}G^\kappa{}_{\tau\rho\eta} = -F_{\mu\sigma} \tag{3.9}
$$

Since $G^\kappa{}_{\tau\rho\sigma}$ always appears contracted with something which is totally antisymmetric in $\tau\rho\sigma$, we may just as well assume that $G^\kappa{}_{\tau\rho\sigma}$ itself is totally antisymmetric in $\tau\rho\sigma$. Given all this, we find that

$$
[\delta_\eta, \delta_\epsilon]A_\mu = 2i\bar{\epsilon}\Gamma^\sigma\eta F_{\sigma\mu} - i\bar{\epsilon}\Gamma^\sigma\eta\left(\frac{1}{6}\epsilon_{\mu\sigma\kappa}{}^{\tau\rho\eta}G^\kappa{}_{\tau\rho\eta} + F_{\mu\sigma}\right) \tag{3.10}
$$

and

$$
[\delta_\eta, \delta_\epsilon]\phi^A_\mu = 2i\bar{\epsilon}\Gamma^\sigma\eta D_\sigma\phi^A_\mu - 4id\bar{\epsilon}\Gamma^\sigma\Gamma_B\eta[\phi^A_\mu, \phi^B_\sigma] \tag{3.11}
$$

That is, all the supersymmetry variations close on-shell (modulo a gauge variation).

The abelian self-duality equation on the gauge field in space-time implies the Maxwell equation of motion. In loop space we may take the point of view that $G^{\kappa}{}_{\tau\rho\sigma}$ is just some auxiliary field that has to be related to $F_{\mu\nu}$ in certain ways (as specified above). The equations of motion do not follow from these relations. To get these we must make a supersymmetry variation of the Fermi equations of motion. We then find the Bianchi identity

$$D_{[\mu}F_{\nu\rho]} = 0 \tag{3.12}$$

and the Bose equations of motion

$$\begin{aligned} D^{\mu}F_{\mu\nu} + [\phi^{\mu}_{A}, D_{\nu}\phi^{A}_{\mu}] + \text{fermions} &= 0 \\ D^{\mu}D_{\mu}\phi^{A}_{\nu} - \frac{1}{2}[\phi_{B,\nu}, [\phi^{B}_{\mu}, \phi^{A,\mu}]] + \text{fermions} &= 0 \end{aligned} \tag{3.13}$$

To get these equations we have made use of all the constraints. In all equations we have presented one should notice the resemblance with super Yang-Mills, to which they reduce upon compactification on a circle.

4. Local fields?

We would of course like to get local fields from the loop fields. In the abelian case we should get the well-known abelian tensor multiplet. To this end we adopt the representations given in Eqs (2.1) and (2.2) of the loop fields. We also let

$$G^{\kappa}{}_{\tau\sigma\rho} = \int dC^{\kappa} H_{\tau\sigma\rho} \tag{4.1}$$

which obviously is a realization of the constraint (3.8). The constraint (3.9) then amounts to

$$H_{\mu\nu\rho}(x) = -\frac{1}{6}\epsilon_{\mu\nu\rho\kappa\tau\sigma}H^{\kappa\tau\sigma}(x), \tag{4.2}$$

and inserting this representation of the loop fields into the supersymmetry variations it is easy to see that the supersymmetry variations of the local fields become

$$\begin{aligned} \delta\phi^{A} &= -i\bar{\epsilon}\Gamma^{A}\psi \\ \delta\psi &= \left(\frac{1}{12}H_{\kappa\tau\rho}\Gamma^{\kappa\tau\rho} + \partial_{\mu}\phi^{A}\Gamma^{\nu}\Gamma_{A}\right)\epsilon \\ \delta B_{\mu\nu} &= -i\bar{\epsilon}\Gamma_{\mu\nu}\psi. \end{aligned} \tag{4.3}$$

Of course we tailored our supersymmetry variations so that we would get these well-known transformations (see for instance [3]) for abelian gauge group.

Requiring Wilson surface observables to exist in our theory, we get severe restrictions on the loop fields. We look for a generalization of the abelian loop field

$$A_{\mu}(C) = \int ds B_{\nu\mu}(C(s))\dot{C}^{\nu}(s) \tag{4.4}$$

to the non-abelian case. Notice that with $B_{\mu\nu}$ being antisymmetric, we have the Lorentz covariant transversality constraint

$$\int ds\dot{C}^{\mu}(s)A_{\mu}(s,C)=0. \tag{4.5}$$

This constraint can (and hence should) be taken over to the non-abelian case because it is gauge covariant[4] so it still makes sense to impose this constraint also in the non-abelian case.[5]

Then for the Wilson surface to be reparametrization invariant we must also require [1]

$$[\mathcal{H}(s,t),\mathcal{H}(s',t)]=0 \tag{4.6}$$

where

$$\mathcal{H}(s,t):=n^{\mu}(s)A_{\mu}(s,C) \tag{4.7}$$

Here $n^{\mu}(s)$ is defined (using the induced metric on the Wilson surface) as the unit vector which is orthogonal to the tangent vector $\dot{C}^{\mu}(s)$. We now look for representations of this loop field $A_{\mu}(C)$ in terms of local fields. The only solution that we have found to these conditions is

$$A_{\mu}(C)=\int dsB^{a}_{\mu\nu}(C(s))\dot{C}^{\nu}(s)T_{a}(s) \tag{4.8}$$

where we let the generators $T_a(s)$ obey the Lie algebra

$$[T_a(s),T_b(s')]=C_{ab}{}^{c}\chi(s-s')T_c(s). \tag{4.9}$$

Here

$$\chi(s)=\delta_{s,0} \tag{4.10}$$

is the function which is 0 everywhere except at $s=0$ where it is 1. It is apparent that condition (4.6) is obeyed. For condition (4.5) to be obeyed $B_{\mu\nu}$ must be taken to be antisymmeric. However, the local theory described by this two-form will be abelian because

$$[T_a,T_b]=0 \tag{4.11}$$

where

$$T_a:=\int dsT_a(s). \tag{4.12}$$

One may also be tempted to try with

$$A_{\mu}(C)=\int dsA^{a}_{\mu}(C(s))\lambda_a(s) \tag{4.13}$$

---

[4] $\delta\int dsA_{\mu}(s,C)\dot{C}^{\mu}(s)=\int ds\left(\dot{C}^{\mu}(s)\frac{\delta}{\delta C^{\mu}(s)}\Lambda(C)+[\dot{C}^{\mu}(s)A_{\mu}(s,C),\Lambda(C)]\right)\equiv 0$

[5] It should also be consistent with SUSY: $\dot{C}^{\mu}\delta A_{\mu}=-i\bar{\epsilon}\Gamma_{\kappa\mu}\psi^{\kappa}\dot{C}^{\mu}$. Thus we find the non-abelian constraint $\psi^{[\mu}\dot{C}^{\nu]}=0$, which obvious holds in the abelian case with the loop fields represented as in eq. (2.2).

JHEP01(2006)165

However there is no way we could satisfy condition (4.5) with this ansatz.

Using the formalism of 2-groups to represent the loop fields in terms of a local two-form and one-form gauge connection has also led to an abelian theory [5]. In conclusion, the issue of finding the appropriate representation for these loop fields seems to be rather tricky. The worst of scenarious would be if it turned out to be impossible to represent them in terms of local fields, unless of course one is interested in just the abelian theory.

A. Spinor conventions

We use the same conventions as [3], that is, we use eleven-dimensional gamma matrices Γ^M and make the split $\Gamma^M = (\Gamma^\mu, \Gamma^A)$ corresponding to the split $\mathrm{SO}(1,10) \to \mathrm{SO}(1,5) \times \mathrm{SO}(5)$. We define

$$\Gamma := \Gamma^{012345}. \tag{A.1}$$

The (anti-)commutation relations between all these gamma matrices are

$$\begin{aligned} \{\Gamma^\mu, \Gamma^\nu\} &= 2\eta^{\mu\nu} \\ \{\Gamma^\mu, \Gamma^A\} &= 0 \\ \{\Gamma^A, \Gamma^B\} &= 2\delta^{AB} \\ \{\Gamma^\mu, \Gamma\} &= 0 \\ [\Gamma^A, \Gamma] &= 0 \end{aligned} \tag{A.2}$$

We impose the following $\mathrm{SO}(1,10)$-invariant Majorana condition on the spinors,

$$\bar{\psi} = \psi^T C. \tag{A.3}$$

Here $\bar{\psi} := \psi^\dagger \Gamma^0$ and the eleven-dimensional charge conjugation matrix C has the properties

$$\begin{aligned} C^T &= -C \\ C^\dagger C &= 1 \end{aligned} \tag{A.4}$$

. Letting V denote the linear space of such Majorana spinors, we then define the $\mathrm{SO}(1,5) \times \mathrm{SO}(5)$-invariant chiral subspaces

$$V_\pm := \{\psi \in V : P_\pm \psi = \psi\}.$$

where

$$P_\pm := \frac{1}{2}(1 \pm \Gamma) \tag{A.5}$$

As a consequence of (A.2), $\Gamma^\mu : V_\pm \to V_\mp$ and $\Gamma^A : V_\pm \to V_\pm$.

The gamma matrices have the properties

$$\begin{aligned} \left(\Gamma^M\right)^T &= -C\Gamma^M C^{-1} \\ \left(\Gamma^{M_1 \cdots M_p}\right)^T &= (-1)^{\frac{p(p+1)}{2}} C\Gamma^{M_1 \cdots M_p} C^{-1} \\ \Gamma^T &= -C\Gamma C^{-1} \end{aligned} \tag{A.6}$$

and

$$\Gamma\Gamma_{\mu\nu\rho} = \frac{1}{6}\epsilon_{\mu\nu\rho\kappa\tau\sigma}\Gamma^{\kappa\tau\sigma}$$

If $\epsilon, \eta \in V$ then we get

$$\bar{\eta}\Gamma_{M_1}\cdots\Gamma_{M_p}\epsilon = (-1)^p\bar{\epsilon}\Gamma_{M_p}\cdots\Gamma_{M_1}\eta \tag{A.7}$$

We will let SUSY parameters be $\epsilon_-, \eta_-, \ldots \in V_-$. The spinor ψ_+ which is in the corresponding tensor multiplet will be of opposite chirality to that of the SUSY parameter, thus $\psi_+ \in V_+$.

We have that

$$\begin{aligned} \bar{\eta}_-\Gamma^{\mu_1}\cdots\Gamma^{\mu_k}\epsilon_- &= 0 \text{ if } k \text{ is even} \\ \bar{\eta}_-\Gamma^{\mu_1}\cdots\Gamma^{\mu_k}\psi_+ &= 0 \text{ if } k \text{ is odd} \end{aligned}$$

To see this we note that $\Gamma\epsilon = \epsilon \Leftrightarrow \bar{\epsilon}\Gamma = -\epsilon$.

In eleven dimensions a complete set of matrices is

$$\left\{1, \Gamma^M, \ldots, \Gamma^{M_1M_2M_3M_4M_5}\right\} \tag{A.8}$$

because $\Gamma^{012\cdots10} = 1$. The number of independent matrices is 2^{10} which is the number of components in a squarical matrix acing on the 2^5-dimensional Dirac spinor representation. In six dimension we take as the complete set the matrices

$$\left\{\Gamma^{\mu_1\cdots\mu_k}\Gamma^{A_1\cdots A_l}\right\} \tag{A.9}$$

in such a way that

$$\sum_{k,l}\binom{6}{k}\binom{5}{l} = 2^{10}. \tag{A.10}$$

A particularly nice choice[6] is to let $k = 0, \ldots, 6$ and $l = 0, 1, 2$.

Using the completeness and normalization properties[7] of these matrices we may obtain the Fierz rearrangement

$$\epsilon\bar{\eta} - \eta\bar{\epsilon} = \frac{1}{16}\left(-(\bar{\eta}\Gamma_\eta\epsilon)\Gamma^\eta + (\bar{\eta}\Gamma_\eta\Gamma_A\epsilon)\Gamma^\eta\Gamma^A\right)(1+\Gamma) - \frac{1}{192}(\bar{\eta}\Gamma_{\mu\nu\rho}\Gamma_{AB}\epsilon)\Gamma^{\mu\nu\rho}\Gamma^{AB} \tag{A.11}$$

Here are some gamma matrix identities we have made use of,

$$\begin{aligned} \Gamma^{\mu\nu}\Gamma^\eta\Gamma_\mu &= 8\eta^{\nu\eta} - 3\Gamma^\eta\Gamma^\nu \\ \Gamma^{\mu\nu}\Gamma^\eta &= \Gamma^\eta\Gamma^{\mu\nu} - 4\eta^{\eta[\mu}\Gamma^{\nu]} \\ \Gamma^{\nu\mu}\Gamma_{\eta\omega\tau}\Gamma_\mu &= \Gamma_{\eta\omega\tau}\Gamma^\nu \\ \Gamma_{AB}\Gamma^{CD}\Gamma^A &= -4\delta_B^{[C}\Gamma^{D]} \\ \Gamma_A\Gamma^{CD}\Gamma^A &= \Gamma^{CD} \end{aligned} \tag{A.12}$$

[6]We notice that $\binom{5}{0} + \binom{5}{1} + \binom{5}{2} = 2^4$ and that $2^6.2^4 = 2^{10}$.

[7]Completeness of a set of matrices Γ^A means that any matrix M may be expanded as $M = \sum C_A\Gamma^A$ and a normalization property $tr(\Gamma_A\Gamma^B) = \delta_A^B$ enable us to determine the coefficients $C_A = tr(M\Gamma_A)$.

JHEP01(2006)165

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JHEP01(2006)165